Crystallographic Realization of the Mathematically Predicted Densest All-Pentagon Packing Lattice by C_5-Symmetric "Sticky" Fluoropentamers**

Changliang Ren, Feng Zhou, Bo Qin, Ruijuan Ye, Sheng Shen, Haibin Su, and Huaqiang Zeng

Two-dimensional periodic and aperiodic tilings have continuously garnered great interest from ancient[1a,b] to present[1c–i] times. Fascinated by the aesthetically pleasing fivefold symmetry and pentagonal shape that are found abundantly from macroscopic morning glories and star fruits, to microscopic virus capsids,[2a] simple liquids,[2b] and to nanoscopic fullerenes, and biological pentamers (Figure 1a),[2c] a fundamental curiosity that has inspired a long-standing quest[1a–c,2a,3] concerns two-dimensional (2D) packing by non-overlapping equal regular pentagons that cannot fully tile a 2D plane without leaving gaps (Figure 1b).

Mathematically, equal regular pentagons can form abstract compact 2D crystal lattices (Figure 1b).[1a,b,2a,3a–c]
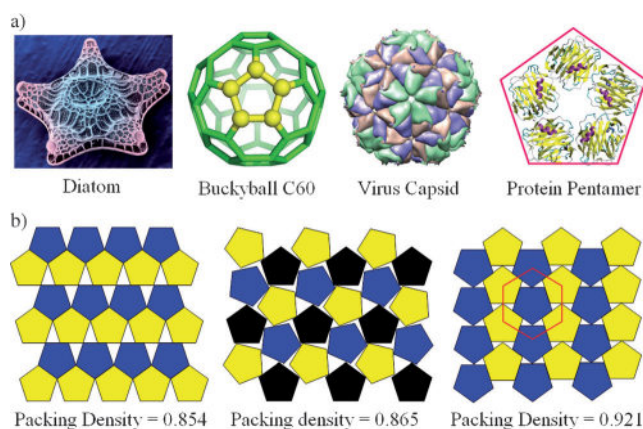

Figure 1. a) Selected examples of the fivefold symmetry or pentagonal shapes that are found abundantly in the world. b) Three mathematically constructed periodic 2D packing patterns and packing densities by non-overlapping equal regular pentagons in the plane. The densest packing containing hexagonally arranged pentagons has a packing density of 0.921.

[*] C. L. Ren, Dr. B. Qin, R. J. Ye, S. Shen, Prof. H. Q. Zeng
   Department of Chemistry, National University of Singapore
   3 Science Drive 3, Singapore 117543 (Singapore)
   E-mail: chmzh@nus.edu.sg

   F. Zhou, Prof. H. B. Su
   Division of Materials Science, Nanyang Technological University
   50 Nanyang Avenue, Singapore 639798 (Singapore)

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With a 2D packing density of 0.921, the periodic densest packing configuration of pentagons in the plane is built of alternating striped rows of pentagons pointing in opposite directions. This leads to a congruent arrangement of hexagonally coordinated pentagons that forms an ingenious solution to overcoming the symmetry mismatch between pentagons and lattice, and creates a 2D packing lattice with *p*2*mg* symmetry. This densest six-neighbor lattice packing was first conjectured by Henley in 1986[4] and mathematically substantiated by Kuperberg and Kuperberg in 1990.[3c]

However, till now experimental observations on pentagonal small molecules,[1g,h,5a] proteins,[5b,c] and macroscopic objects,[1e,5g] as well as computational simulations,[5d–f] all pointed to a great difficulty for regular pentagons to form a densely packed crystal lattice in flat 2D space. In the instances where pentagons appear to be densely packed in some areas,[1g,5c,g] disorders/defects of varying types (orientational/translational disorder, lattice dislocation, etc.) exist. On rare occasions, dense packing of pentagons with the highest packing density of 0.921 could be found in limited areas by experiments on macroscopic objects[5g] and by computational simulations on hard pentagons.[5h]

A few recent studies elegantly explored the use of fivefold-symmetric corannulene and its derivatives to look into pentagonal packing diagrams on copper surface. A low packing density of 0.765 for ordered 2D packing by pentagons[1g] and surface-induced trimerization of pentagons with a packing density of less than 0.725[1h] were obtained. Therefore, down to the molecular level, whether or not and how regular pentagons of uniform size can pack ultradensely in the 2D plane are still open questions that have remained elusive from our current understanding of matter and that have been hampered by the insufficient diversity of pentagonal molecules,[1h,2c,3b,6,7] and especially by the scarce availability of neutral molecules having fivefold symmetric planarity.[7a]

In our quest for possible means of experimentally reproducing the densest mathematically predicted crystals at the molecular level, we envisioned that the key to better understanding and possibly solving the periodic packing problem faced by regular pentagons lies in our ability to produce fivefold-symmetric, planar molecules that resemble the overall shape of a hard pentagon. To this end, we set out to engineer a pentagonal molecule that ideally should have unique fivefold-symmetric planarity. Following a cue given by pentagonal molecule **1** (Figure 2), recently reported by us,[2c] and the proven ability of aromatic fluorine atoms to form intramolecular H bonds that effectively constrain the aromatic backbones into crescent or helical shapes,[8] aromatic
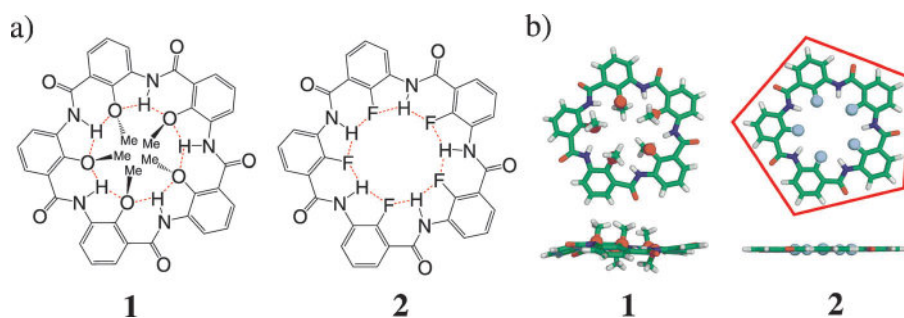
Figure 2. a) Chemical structures of **1** and **2**. In **1**, three methyl groups point up and two point down. Dotted red lines indicate H bonds. b) Top and side views of crystal structure of **1** and computationally optimized structure of **2** (B3LYP/6-31G* level). Red spheres: O atoms, cyan spheres: F atoms. "Sticky" edges and vertices in **2** are made up of an exterior array of partially charged aromatic hydrogen (H-bond donor) and carbonyl oxygen atoms (H-bond acceptor).

fluoropentamer **2** (Figure 2) was finally conceived as a conceptual pentameric molecular framework to explore its possible shape, molecular symmetry, and solid-state packing. Its crystal structure was used to realize the densest planar pentagons with perfect long-range order at the atomic scale that was previously thought possible by physicists and mathematicians.[2a, 3a,b]

As a first step toward deriving the topography associated with **2**, we constructed ab initio computational molecular model using DFT at the B3LYP/6-31G(d,p) level, which has consistently enabled us to accurately predict 3D topographies of oligomers of varying types that were later experimentally verified by crystal structures.[2c, 9] As shown in Figure 2b, the geometry of the optimized structure of **2** shows fivefold-symmetric planarity, and the extent of backbone bending is insignificantly influenced by covalent macrocyclization constriction.[2c, 9a,b] Instead, its crescent-shaped backbone curvature is primarily induced and maintained by an inward-pointing, continuous C−F···H−N H-bonded network. Such an intrinsic backbone peculiarity requiring five identical repeating units to form a macrocycle is rarely observed in other systems.[6] Moreover, the exterior array of partially charged aromatic hydrogen (H-bond donor) and carbonyl oxygen atoms (H-bond acceptor) may serve as "sticky" atoms to glue the molecules together through intermolecular C−H···O=C H bonds (Figure 2a).

To further characterize our design, pentamer **2** was synthesized (see Scheme S1 of Supporting Information) to test the ab initio model and shed new light on the old mystery of 2D pentagons assembled with long-range order. Compound **2** was unambiguously characterized by HRMS and ¹H NMR and ¹⁹F NMR spectroscopy (see Figures S1 and S2 of Supporting Information). Evidence that short oligomers such as dimer **2c** and trimer **3f** adopt crescent-shaped conformations enforced by intramolecular C−F···H−N H bonds is provided in Table S2 and Figures S3 and S4 of the Supporting Information.

To investigate the symmetry and solid-state assembly of **2** in the 2D plane, single crystals of **2** were eventually obtained by slow cooling of a solution containing about 1.1 mg mL⁻¹ of **2** in hot DMSO from 110 to 25 °C over

three months. The determined crystal structure[10] demonstrates folding of **2** into an almost planar disk arrangement of nearly perfect fivefold symmetry in solid state (Figure 3 a,b and Table S5 of Supporting Information). All five fluorine atoms and amide protons point inward, and contribute to formation of a continuous intramolecularly H-bonded network (CF···HN 2.08–2.26 and 1.94–2.27 Å for S(5) and S(6) intramolecular H bonds, respectively). This H-bonding network provides the required entropically favorable preorganization force to rigidify the amide linkages and instruct an otherwise randomly oriented backbone to fold into a well-defined, rigid, circularly folded conformation with pentagonal shape. Five fluorine atoms convergently align to enclose a star-shaped cavity of about 2.85 Å in radius from the cavity center to the center of the fluorine nucleus (Figure 3 a,b).

Analysis of the solid-state packing pattern in **2** reveals the long-awaited densest striped crystalline lattice (Figure 3 e) harmonically built from pure pentagonal molecules that pack precisely in the same manner as the mathematically verified
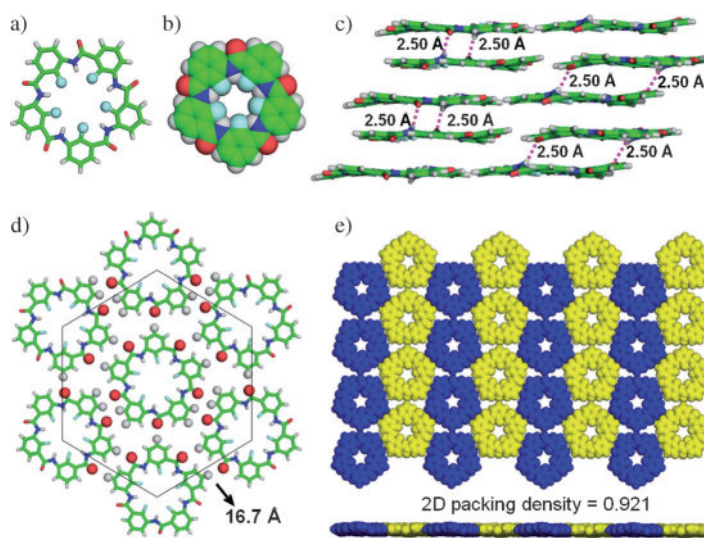

Figure 3. Crystal structure and 2D molecular packing of macrocyclic fluoropentamer **2**. a) Top view. b) CPK model of **2** built on the basis of van der Waals radii (H gray: 1.20; C green: 1.70; O red: 1.52; N blue: 1.55; F cyan: 1.47 Å). c) Formation of discrete pairs of isolated H bonds (2.50 Å) between two pentamers, which stabilize the interplanar stacking. d) Pseudo-hexagonal arrangement with a side length of 16.7 Å and with each pentagon forming two edge-to-vertex and two smaller and two larger edge-overlap contacts with its six close neighbors. Small spheres represent partially charged "sticky" hydrogen (gray) and oxygen (red) atoms that are involved in C=O···H−C H bonds, which allow tight packing of pentagonal molecules. e) Top and side views of alternating striped lattice packing, consisting of pentagons pointing in opposite directions in CPK model, which reproduces the densest mathematical crystal formed by pentagons (Figure 1 b). The average interplanar distance is 3.1 Å. The 2D packing density of 0.921 was calculated by using the van der Waals radii specified in (b).

six-neighbor densest packing (Figure 1b).[3a,c] While each parallelogramic unit cell contains two pentamers, the closest packing pattern can be better represented by the translationally ordered pseudo-hexagonally coordinated pentamers (Figure 3 d,e). These pentamers having no vertices but segments of their sides in common are arranged to form alternating stripes that point head-to-tail in opposite directions. Each pentamer has five complementary "sticky" edges and one vertex to attract six close neighbors by means of H bonding of three types: two edge-to-vertex, two smaller edge-overlap, and two larger edge-overlap bonding contacts. All of these specific H-bonding interactions are mediated by weak C−H···O=C H-bonding forces. Because of these specific H bonds conserved among pentamers that continue across the 2D lattice, minimization of the gaps among "sticky" pentamers in the 2D plane (Figure 3 d,e) is as good as in the mathematically conceived packing lattice (Figure 1b). Thus our crystal has an experimentally determined 2D packing density of 0.921, a value theoretically predicted to be the highest possible.[2a,3a,c]

Driven by a combination of aromatic π–π stacking interactions and numerous interplanar H bonds (C=O···H−N 2.50 Å, Figure 3c), the ordered 2D lattices further crystallize into a 3D structure. One pair of isolated interplanar H bonds creates a discrete dimeric ensemble composed of two pentamers (Figure 3c). These interplanar H bonds result in an observed average interplanar distance as short as 3.1 Å and a high crystal density of 1.58 $Mg m^{-3}$. They form as a result of comparatively much weaker C−F···H−N H bonds causing two amide bonds in each pentamer to twist out of the pentameric plane to form stronger intermolecular C=O···H−N H bonds (2.50 Å) with the adjacent neighboring pentamer. Such a twist causes pentamers to deviate from the idealized fivefold-symmetric planarity of the computed structure of **2** (Figure 2b), and changes the 2D lattice symmetry from $p2$ to pseudo-$p2$.

To eliminate the above interplanar crystal-packing effect that gives rise to distorted amide bonds, and thus obtain not only the perfect packing pattern and packing parameters for perfect regular pentagons, but also quantitative information on driving forces responsible for the formation of the ordered 2D pentagonal lattice, we continued our theoretical investigation at the B3LYP/6-31G(d,p) level on the hexagonal unit comprising one central pentamer and six surrounding pentamers (Figure 3d and Supporting Information). A key finding of this theoretical treatment is that the 2D packing lattice formed by these pentagons is perfectly coplanar (see Figure S8a of Supporting Information) in the absence of interplanar layer-to-layer H-bonding interactions as seen in Figure 3c. Further computational analysis revealed driving forces accounting for the bonding energy of -6.82 kcal mol^{-1} per pentamer in forming a 2D pentagonal lattice from regular pentagons, that is, the stabilizing H-bonding interactions between "sticky" hydrogen and oxygen atoms (-14.05 kcal mol^{-1} per pentamer; Table S6 of Supporting Information) can more than compensate for the repulsive interactions among hydrogen atoms and among oxygen atoms, which were estimated to destabi-

lize the 2D packing lattice by 5.33 and 2.50 kcal mol^{-1} per pentamer, respectively (Table S6 of Supporting Information).

Given its nearly planar pentagonal shape with an exterior array of "sticky" edges and vertices, macrocyclic pentamer **1** (Figure 2) should also be capable of assembly into a 2D lattice with long-range order and translational symmetry in the plane. Re-examining the crystal structure of **1**[2c] indeed reveals a periodic 2D six-neighbor packing pattern (Figure 4), remarkably similar to the densest pentagonal packing of **2** (Figure 3e). The main driving force for organizing the interplanar stacking is van der Waals interactions among the protruding methoxymethyl groups. As a result, the average interplanar distance increases substantially to 4.3 Å, 0.9 Å larger than the typical distance of 3.4 Å, and the crystal density decreases to 1.43 Mg m^{-3}. In contrast to **2**, the 2D lattice built from **1** consists of slightly distorted hexagonal units that tile the plane, all of which contain seven water molecules that each participate in forming one strong intermolecular HO−H···O=C H bond (2.01 Å) with the closest pentamer molecule. To accommodate these water molecules in the lattice, 1) the gaps among pentamers become larger, reducing the 2D packing density of pentamers to 0.898 with a further contribution to packing density of 0.035 from seven water molecules (Figure 4b), and 2) each pentamer has two edge-to-vertex, one smaller and one larger edge-overlap, and one full edge contacts with its five close neighbors, while the sixth neighbor interacts with it through two water molecules.

Comparison of 2D molecular packing between **1** and **2** highlights the important fact that, even for rigid pentagonal molecules, a great degree of adaptive flexibility in crystal packing in response to external impurities such as water molecules can be achieved. The fact that such physically realistic lattices do exist suggests that molecules with fivefold rotational symmetry are fully compatible with the symmetry and translational ordering of 2D crystal lattices, and can pack with high density. In addition to idealized $p2mg$ symmetry
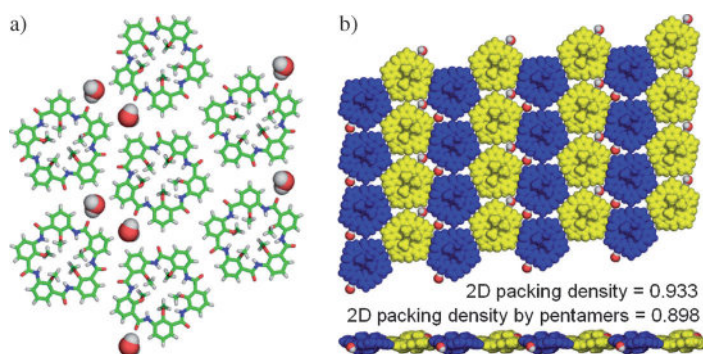

2D packing density = 0.933
2D packing density by pentamers = 0.898

Figure 4. 2D molecular packing of macrocyclic pentamer **1**. a) Distorted hexagonal unit consisting of seven pentamers and seven water molecules (CPK models), each of which forms a strong intermolecular HO−H···O=C H bond (2.01 Å) with its closest neighbor. b) Top and side views of the alternating striped lattice packing as CPK model, consisting of water molecules and pentagons pointing in opposite directions. In contrast to the packing pattern in Figure 3e, one pentagonal edge of the pseudo-hexagonally coordinated central pentagon interacts with one of its close neighbors through two water molecules. The average interplanar distance is 4.3 Å. The 2D packing densities of 0.933 and 0.898 were calculated on the basis of van der Waals radii specified in the legend to Figure 3b.

(Figure 1 b) and depending on the molecular structures and their packing patterns, the 2D lattices built from pure pentagonal tiles can have symmetries of $p2$ for 2D packing by **2** (Figure 3 e), and $p1$ for 2D packing by **1** (Figure 4 b).

Our above design strategy additionally affords covalent nanoscaffolding for the expedient exchange of a multitude of modular functional groups that decorate both the exterior and interior surfaces of the macrocyclic pentamers. This could lead to a large diversity of freely standing pentagonal molecules that offer interesting perspectives on other mathematically sketched periodic pentagonal tiling patterns (Figure 1 b)[1a,b,2a,3a–c] whose 2D crystalline packings at the molecular scale still remain to be found. The pentagonal shape of the molecules, their validated pentagon packing models, and interaction mechanism through complementary "sticky" edges and vertices may also become useful for nanosurface patterning of novel materials.[1d,g,h,11]

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[10] The obtained values of $R1$ (0.1495) and $wR2$ (0.2957) are larger than expected, which is presumably due to the disorder as a result of two orientations of the molecule overlapping in one asymmetric position. The two disordered parts represent different directions in which the amide groups go around the pentamer ring in (anti)clockwise fashion. Since the atoms of the two disordered parts are largely overlapping, large numbers of strict restraints were applied to the refinement so that it became convergent and the structure remained reasonable. In the refinement, we used the disorder scheme in which all atoms are involved in the disorder so that the geometry of the disordered amides is not distorted. This explains the unusually high values of the final R factors. Without introducing disorder, the result was even worse. At the same time, we refined the structure by changing the weight scheme and omitting several reflections, all of which help to decrease R factors, correct the GOF, and remove high residual peaks. The ratio of the disordered parts, as obtained by free variable refinement, is about 56:44, which is close to the random ratio of 50:50. From the structural refinement, the existence of a five-membered ring is clear.

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